(Time: $2\frac{1}{2}$ hours)

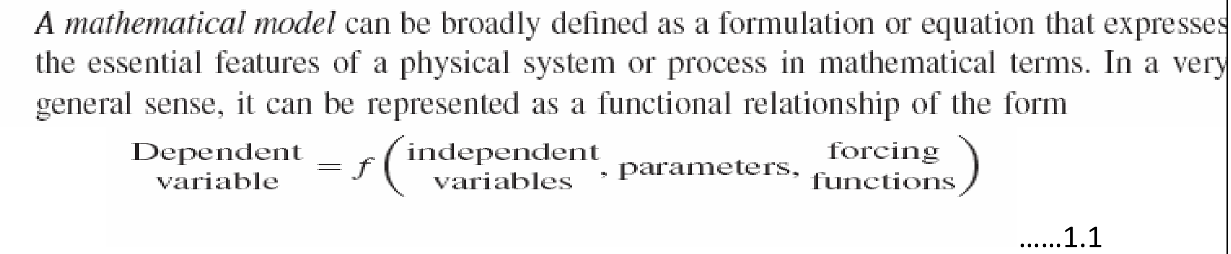
[Marks: 75]

Please check whether you have got the right question paper.

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculator is **allowed**.

1. **Attempt *any three* of the following:** **15**

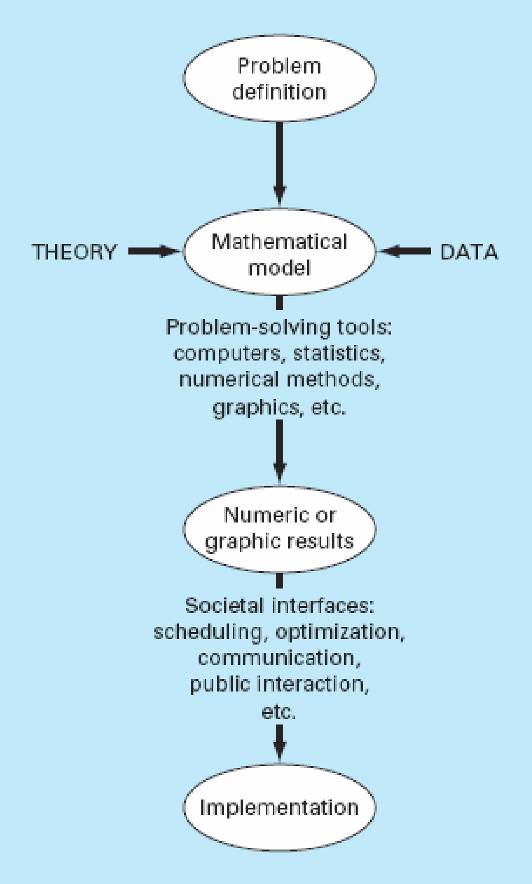
a. What is a mathematical model? With the help of a flow chart , explain the of solving an engineering problem ?



A *mathematical model* can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms. In a very general sense, it can be represented as a functional relationship of the form

$$\text{Dependent variable} = f\left(\text{independent variables}, \text{parameters}, \text{forcing functions}\right)$$

……1.1



where the dependent variable is a characteristic that usually reflects the behavior or state of the system; the independent variables are usually dimensions, such as time and space, along which the system's behavior is being determined; the parameters are reflective of the system's properties or composition; and the forcing functions are external influences acting upon the system.

The actual mathematical expression of Eq. (1.1) can range from a simple algebraic relationship to large complicated sets of differential equations. For example, on the basis of his observations, Newton formulated his second law of motion, which states that the time rate of change of momentum of a body is equal to the resultant force acting on it. The mathematical expression, or model, of the second law is the well-known equation

$F = ma$ …… (1.2)

where F = net force acting on the body (N, or kg m/s^2), m = mass of the object (kg), and a = its acceleration (m/s^2).

b. Create a hypothetical floating-point number set for a machine that stores information using 7-bit words. Employ the first bit for the sign of the number, the next three for the sign and the magnitude of the exponent, and the last three for the magnitude of the mantissa.

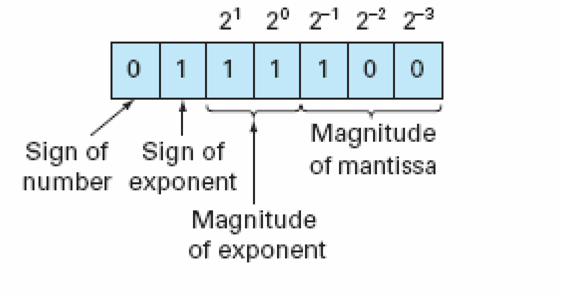
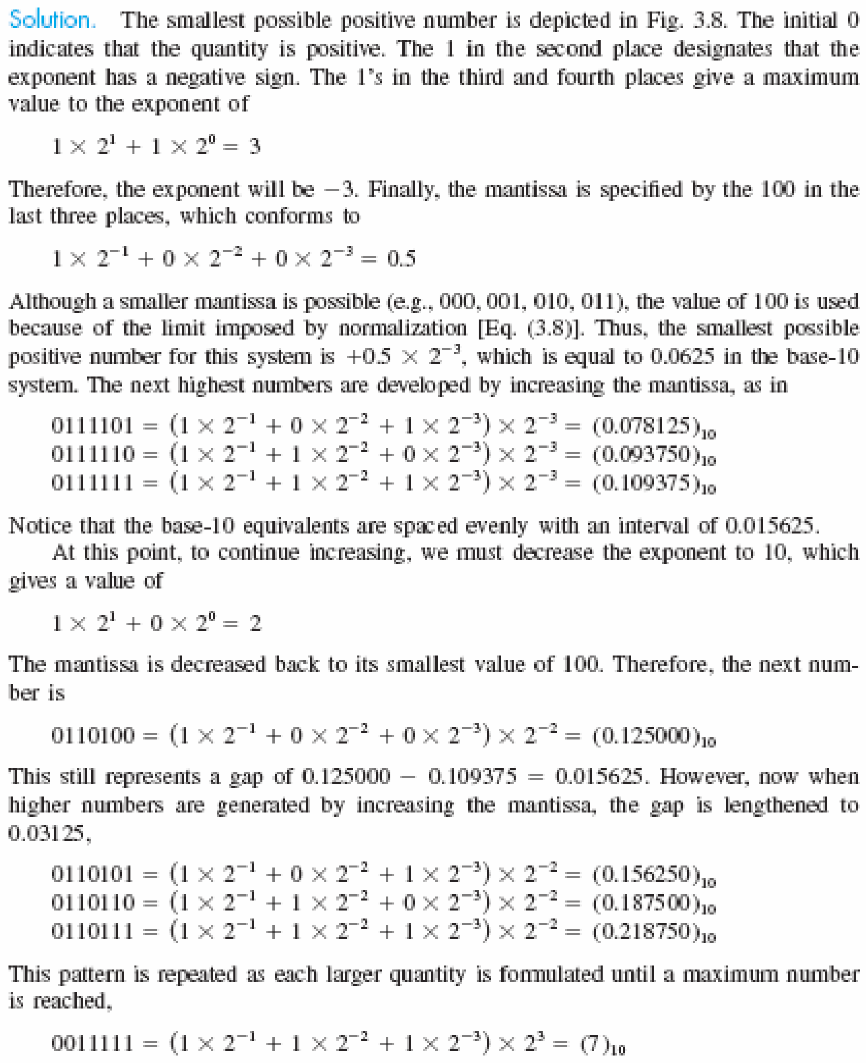


Figure 3.8



Solution. The smallest possible positive number is depicted in Fig. 3.8. The initial 0 indicates that the quantity is positive. The 1 in the second place designates that the exponent has a negative sign. The 1's in the third and fourth places give a maximum value to the exponent of

$$1 \times 2^1 + 1 \times 2^0 = 3$$

Therefore, the exponent will be −3. Finally, the mantissa is specified by the 100 in the last three places, which conforms to

$$1 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3} = 0.5$$

Although a smaller mantissa is possible (e.g., 000, 001, 010, 011), the value of 100 is used because of the limit imposed by normalization [Eq. (3.8)]. Thus, the smallest possible positive number for this system is $+0.5 \times 2^{-3}$, which is equal to 0.0625 in the base-10 system. The next highest numbers are developed by increasing the mantissa, as in

$$0111101 = (1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}) \times 2^{-3} = (0.078125)_{10}$$
$$0111110 = (1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3}) \times 2^{-3} = (0.093750)_{10}$$
$$0111111 = (1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}) \times 2^{-3} = (0.109375)_{10}$$

Notice that the base-10 equivalents are spaced evenly with an interval of 0.015625.

At this point, to continue increasing, we must decrease the exponent to 10, which gives a value of

$$1 \times 2^1 + 0 \times 2^0 = 2$$

The mantissa is decreased back to its smallest value of 100. Therefore, the next number is

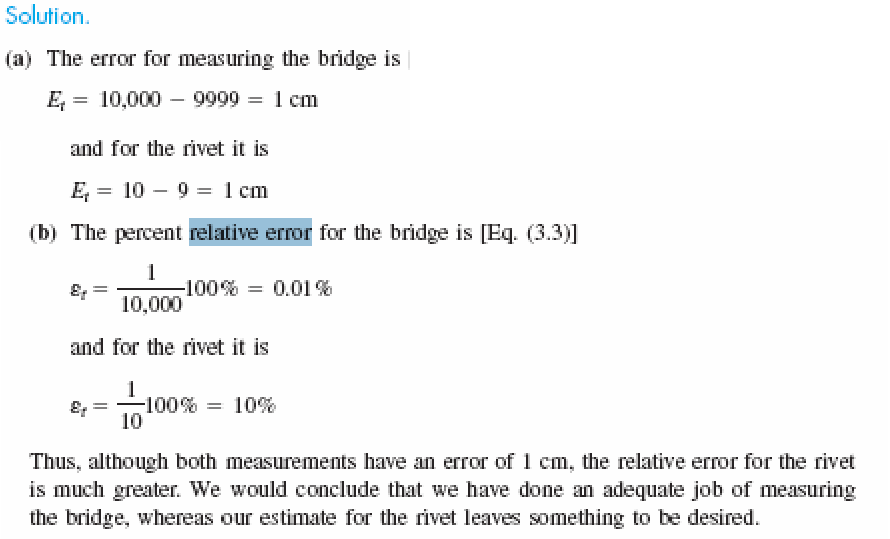
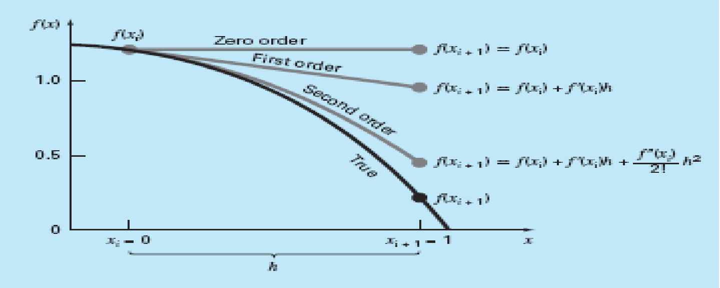
$$0110100 = (1 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3}) \times 2^{-2} = (0.125000)_{10}$$

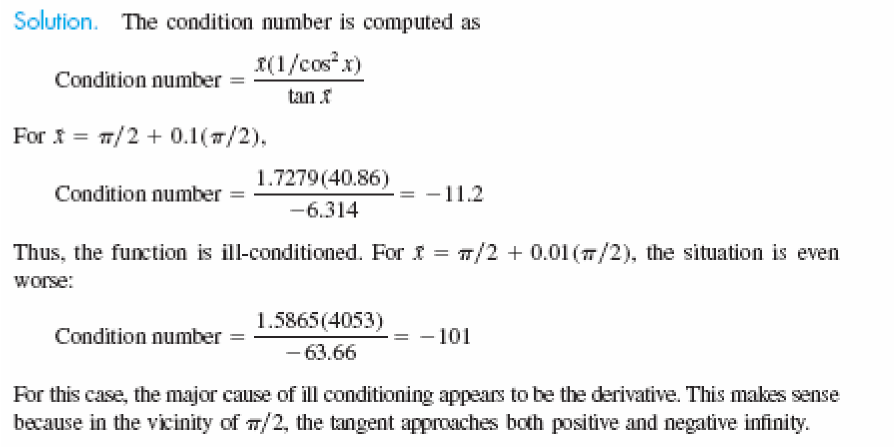
This still represents a gap of 0.125000 − 0.109375 = 0.015625. However, now when higher numbers are generated by increasing the mantissa, the gap is lengthened to 0.03125,

$$0110101 = (1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}) \times 2^{-2} = (0.156250)_{10}$$
$$0110110 = (1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3}) \times 2^{-2} = (0.187500)_{10}$$
$$0110111 = (1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}) \times 2^{-2} = (0.218750)_{10}$$

This pattern is repeated as each larger quantity is formulated until a maximum number is reached,

$$0011111 = (1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}) \times 2^3 = (7)_{10}$$

c.	Suppose that you have the task of measuring the lengths of a bridge and a rivet and come up with 9999 and 9 cm, respectively. If the true values are 10,000 and 10 cm, respectively, compute (i) the true error and (ii) the true percent relative error for each case.  Solution. (a) The error for measuring the bridge is $E_t = 10{,}000 - 9999 = 1$ cm and for the rivet it is $E_t = 10 - 9 = 1$ cm (b) The percent relative error for the bridge is [Eq. (3.3)] $\varepsilon_t = \frac{1}{10{,}000} 100\% = 0.01\%$ and for the rivet it is $\varepsilon_t = \frac{1}{10} 100\% = 10\%$ Thus, although both measurements have an error of 1 cm, the relative error for the rivet is much greater. We would conclude that we have done an adequate job of measuring the bridge, whereas our estimate for the rivet leaves something to be desired.	
d.	Use zero- through fourth-order Taylor series expansions to approximatethe function $f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$ from $x_i = 0$ with h = 1. That is, predict the function's value at $x_{i+1} = 1$. Because we are dealing with a known function, we can compute values for $f(x)$ between 0 and 1. The results indicate that the function starts at $f(0) = 1.2$ and then curves downward to $f(1) = 0.2$. Thus, the true value that we are trying to predict is 0.2. The Taylor series approximation with $n = 0$ is $f(x_{i11}) = 1.2$  	
e.	Compute the condition number for $f(x) = tanx\ for\ \tilde{x} = \frac{\pi}{2} + 0.1\left(\frac{\pi}{2}\right)$ $f(x) = tanx\ for\ \tilde{x} = \frac{\pi}{2} + 0.01\left(\frac{\pi}{2}\right)$	



Solution. The condition number is computed as

$$\text{Condition number} = \frac{\tilde{x}(1/\cos^2 x)}{\tan \tilde{x}}$$

For $\tilde{x} = \pi/2 + 0.1(\pi/2)$,

$$\text{Condition number} = \frac{1.7279(40.86)}{-6.314} = -11.2$$

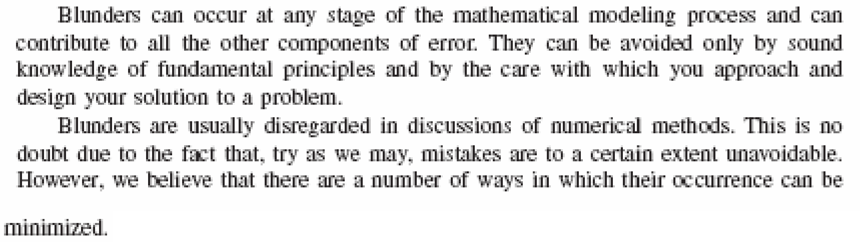
Thus, the function is ill-conditioned. For $\tilde{x} = \pi/2 + 0.01(\pi/2)$, the situation is even worse:

$$\text{Condition number} = \frac{1.5865(4053)}{-63.66} = -101$$

For this case, the major cause of ill conditioning appears to be the derivative. This makes sense because in the vicinity of $\pi/2$, the tangent approaches both positive and negative infinity.

f. Explain blunders, formulation errors and data uncertainty.

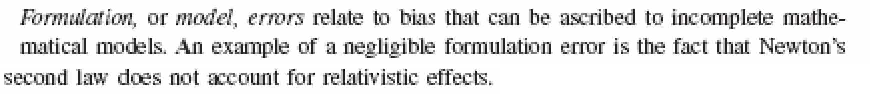
Blunders:



Blunders can occur at any stage of the mathematical modeling process and can contribute to all the other components of error. They can be avoided only by sound knowledge of fundamental principles and by the care with which you approach and design your solution to a problem.

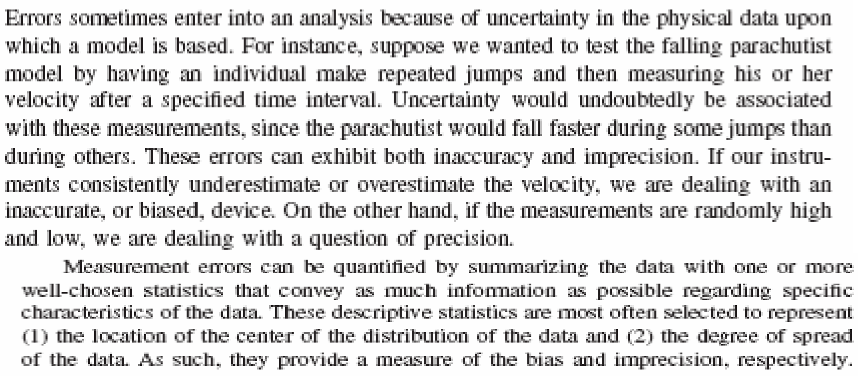
Blunders are usually disregarded in discussions of numerical methods. This is no doubt due to the fact that, try as we may, mistakes are to a certain extent unavoidable. However, we believe that there are a number of ways in which their occurrence can be minimized.

Formulation Errors:



Formulation, or *model, errors* relate to bias that can be ascribed to incomplete mathematical models. An example of a negligible formulation error is the fact that Newton's second law does not account for relativistic effects.

Data Uncertainty:



Errors sometimes enter into an analysis because of uncertainty in the physical data upon which a model is based. For instance, suppose we wanted to test the falling parachutist model by having an individual make repeated jumps and then measuring his or her velocity after a specified time interval. Uncertainty would undoubtedly be associated with these measurements, since the parachutist would fall faster during some jumps than during others. These errors can exhibit both inaccuracy and imprecision. If our instruments consistently underestimate or overestimate the velocity, we are dealing with an inaccurate, or biased, device. On the other hand, if the measurements are randomly high and low, we are dealing with a question of precision.

Measurement errors can be quantified by summarizing the data with one or more well-chosen statistics that convey as much information as possible regarding specific characteristics of the data. These descriptive statistics are most often selected to represent (1) the location of the center of the distribution of the data and (2) the degree of spread of the data. As such, they provide a measure of the bias and imprecision, respectively.

2. Attempt *<u>any three</u>* of the following: 15

a. Find the roots of the equation

$$x^3 - 12.2x^2 + 7.45x + 42 = 0$$

between 11 and 12 using Regula-Falsi method correct upto 4 decimal places.

A

$$\mathbf{c = \frac{a\,f[b] - b\,f[a]}{f[b] - f[a]}}$$

The approximate value of root is 11.20

b. Find the roots of the equation $x \tan x = 1$

near 4 using Newton Raphson method correct up to 4 decimal places.

A

Use the formula

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

Answer **3.4256**

c.

Use the Secant method to find a solution to $x = cosx$ correct up to 4 decimal places.

Use the formula:

$$x_{i+1} = x_i - \frac{f(x_i) * (x_i - x_{i-1})}{f(x_i) - f(x_{i-1})} \qquad i = 1,2,3...$$

Answer: **0.739085**

d.

Given log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990 and log 7 = 0.8451. Find the value of log 47.

A

X_0=40, log40=1+log4=1+2log2=1.6020=y_0
X_1=42,log42=log7+log3+log2=1.6234=y_1
X_2=45,log45=2log3+log5=1.6532=y_2
X=47,log47=…………..=?=y
X_3=49,log49=2log7=1.6902=y_3
X_4=50,log50=1+log5=1.6990=y_4

Applying Lagrange's interpolation formula(where x=47)

$$\log 47 = \frac{(x-x1)(x-x2)(x-x3)(x-x4)}{(x0-x1)(x0-x2)(x0-x3)(x0-x4)}y0 + \frac{(x-x0)(x-x2)(x-x3)(x-x4)}{(x1-x0)(x1-x2)(x1-x3)(x1-x4)}y1 + \frac{(x-x0)(x-x1)(x-x3)(x-x4)}{(x2-x0)(x2-x1)(x2-x3)(x2-x4)}y2$$
$$+ \frac{(x-x0)(x-x1)(x-x2)(x-x4)}{(x3-x0)(x3-x1)(x3-x2)(x3-x4)}y3 + \frac{(x-x0)(x-x1)(x-x2)(x-x3)}{(x4-x0)(x4-x1)(x4-x2)(x4-x3)}y4$$

After substituting the values
Log47 = 1.6721

e.

The table below gives the value of $tan\theta$. Evaluate $tan67°20'$

θ	65°	66°	67°	68°	69°
$tan\theta$	2.1445	2.2460	2.3559	2.4751	2.6051

A

The difference table is

θ	Y = tanθ	Δy	Δ^2y	Δ^3y	Δ^4y
65^0	2.1445	0.1015	0.0884	0.0023	0.0078
66^0	2.2460	0.1099	0.0107	0.0101	
67^0	2.3559	0.1192	0.0208		
68^0	2.4751	0.1300			
69^0	2.6051				

Taking $x_0 = 65^0$, $u = (65^0 20' - 65^0)/1^0 = 67^0.3333 - 65^0 = 2.3333$

$$\therefore \tan 67^0 \cdot 20' = \sum_0^4 \frac{u^r}{r!} \Delta^r y_0$$

$$= 2.1445 + (2.3333)(0.1015) + \frac{(2.1333)(1.3333)}{2!}(0.0084) + \frac{(2.1333)(1.3333)(0.3333)}{3!}(0.0023) + \frac{(2.1333)(1.3333)(0.3333)(-0.6667)}{4!}(0.0078)$$

$$= 2.37296$$

If we take $x_0 = 67^0$, $u = \frac{67.3333-67}{1} = 0.3333$

$$\therefore \tan 67^0.20' = 2.3559 + (0.3333)(0.1192) + \frac{(0.3333)(-0.6667)}{2!}(0.0208)$$

= 2.3932

Correct value is 2.3945

f. From the table of Bessel function $J_n(1)$, estimate the value of $J_{\frac{3}{2}}(1)$

n	-1	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$J_n(1)$	-0.4401	0.0447	0.4311	0.6694	0.7652	0.7522	0.6714	0.5587	0.4401

A Use Newton's backward interpolation formula.

(Ans: 0.2641)

3. Attempt *any three* of the following: 15

a. Solve the following simultaneous equations by Gauss – Jordan elimination method:

$$2x_1 + 6x_2 - x_3 = -14$$

$$5x_1 - x_2 + 2x_3 = 29$$

$$x_3 - 3x_1 - 4x_2 = 4$$

A

$$\begin{bmatrix} 2 & 6 & -1 \\ 5 & -1 & 2 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} -14 \\ 29 \\ 4 \end{bmatrix}$$

Final solution is

x_1=3.8620

X_2==-3.0689

X3=3.3103

b. Solve the following simultaneous equations by Gauss – Seidel method:

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$

$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$

$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

A X1=(7.85+0.1x2+0.2x3)/3

X2=(-19.3-0.1x1+0.3x3)/7

X3=(71.4-0.3x1+0.2x2)/10

First iteration:

X1=2.6166

X2=-2.7945

X3=7.0056

Second Iteration:

X1=2.9905

X2=-2.4996

X3=7.0002

c. For the set of points (0, 2), (2, -2), (3, -1), evaluate $\left(\frac{dy}{dx}\right)_2$

A Since the points are unevenly spaced, we have by Lagrange's formula,

Y=(2) $\frac{(x-2)(x-3)}{(-2)(-3)}$+$\frac{(x-0)(x-3)}{(2)(-1)}(-2)$+ $\frac{(x-0)(x-2)}{(3)(1)}$(-1)

= $\frac{1}{3}$(6x-12)

= 2x-4

$\therefore \left(\frac{dy}{dx}\right)_2$=2(2)-4

=0

d. Evaluate $\int_1^2 \frac{1-e^{-x}}{x}dx$ using trapezoidal rule and Simpson's 3/8 rule.

Let n =5

Hence h = $\frac{(2-1)}{5}$ = 0.2

X0=1	1.2	1.4	1.6	1.8	2.0
Y0=0.6321	Y1=0.5823	Y2=0.5381	Y3=0.4988	Y4=0.4637	Y5=0.4323

Trapezoidal Rule

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n],$$

= 0.51674

By Simpson’s 3/8 th rule = 0.5110725

e. Solve $\frac{dy}{dx} = x + y$; y(1) = 1 for the interval 1 (0.1) 1.2, using method of Taylor series.

A
Y0=x0=1
X1=1.1 y1=?
X2=1.2 y2=?
h=0.1
y’=x+y $y'_0=2$
y”=1+y’ $y''_0=3$
y”’=y” $y'''_0=3$
y^{iv}=y”’ $y^{iv}_0=3$
hence by Taylor series
$y(1.1)=y_0+\frac{hy'_0}{1}+\frac{hy''_0}{2!}+\frac{hy'''_0}{3!}+\frac{hy^{iv}_0}{4!}+\ldots\ldots$
Y(1.1)=1.2155
Y(1.2)=1.3486

f. Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, where y(0) = 1, to find y(0.1) using Runge-Kutta method.

A
$f(x,y)=\frac{y-x}{y+x}$
$x_0=0$ $y_0=1$
h=0.1
k_1=h.f(x0,y0)= 0.1f(0,1)
k_1=0.1
$k_2=hf(x0+\frac{h}{2},y0+k_1)$
k_2=0.1 f(0.05,0.2)
k_2=0.06
$k=\frac{1}{2}(k_1+k_2)$
k=0.08
y_1=y0+k
y_1=1+0.08
y_1=1.08 at x_1=0.1

4. **Attempt *any three* of the following:** **15**

a Fit a straight line to the x and y values in the two rows:

x	1	2	3	4	5	6	7
y	0.5	2.5	2.0	4.0	3.5	6.0	5.2

A

M=7 which is odd

Let three required set line for best fit be

Y=a+bx ………...1

Normal equations are

$\sum y=ma+b\sum x$ …….2

$\sum xy=a\sum x+b\sum x^2$ …..3

Consider the table

no

x	Y	xy	X^2
1	0.5	0.5	1
2	2.5	5.0	4
3	2.0	6.0	9
4	4.0	16.0	16
5	3.5	17.5	25
6	6.0	36.0	36
7	5.2	36.4	49
$\sum x=28$	$\sum y=23.7$	$\sum xy=118$	$\sum X^2=140$

Substituting in equation (2) & (3) we get

23.7=7a+28b

118.4=28a+140b

Solving we get

a=0.0145

b=0.8428

Required straight line is

y=0.0145+0.8428x

b

Fit a second degree parabola for the following:

x	2.5	3	3.5	4	4.5	5	5.5
y	4.32	4.83	5.27	5.47	6.26	6.79	7.23

The equation of second degree parabola is given by

$Y=a+bx+cx^2$

Normal equations are

$\sum y=ma+b\sum x+c\sum x^2$ …………….(2)

$\sum xy=a\sum x+b\sum x^2+c\sum x^3$ ……………(3)

$\sum x^2y=a\sum x^2+\sum x^3+c\sum x^4$ ………..(4)

Here m=7

x	y	X^2	X^3	X^4	xy	X^2y
2.5	4.32	6.25	15.625	39.0625	10.8	27
3	4.83	9	27	81	14.49	43.47
3.5	5.27	12.25	42.875	150.0625	18.445	64.5575
4	5.47	16	64	256	21.88	87.52
4.5	6.26	20.25	95.125	410.0625	28.17	126.765
5	6.79	25	125	625	33.95	169.75
5.5	7.23	30.25	166.375	915.0625	39.765	218.7075

$\sum x=28$ $\sum y=40.17$ $\sum x^2=119$ $\sum x^3=532$ $\sum x^4=2476.25$ $\sum xy=167.5$ $\sum x^2y=737.77$

Substituting in 2,3,4 we get

40.17=7a+28b+119c

167.5=28a+119b+532c

737.77=119a+532b+2476.25c

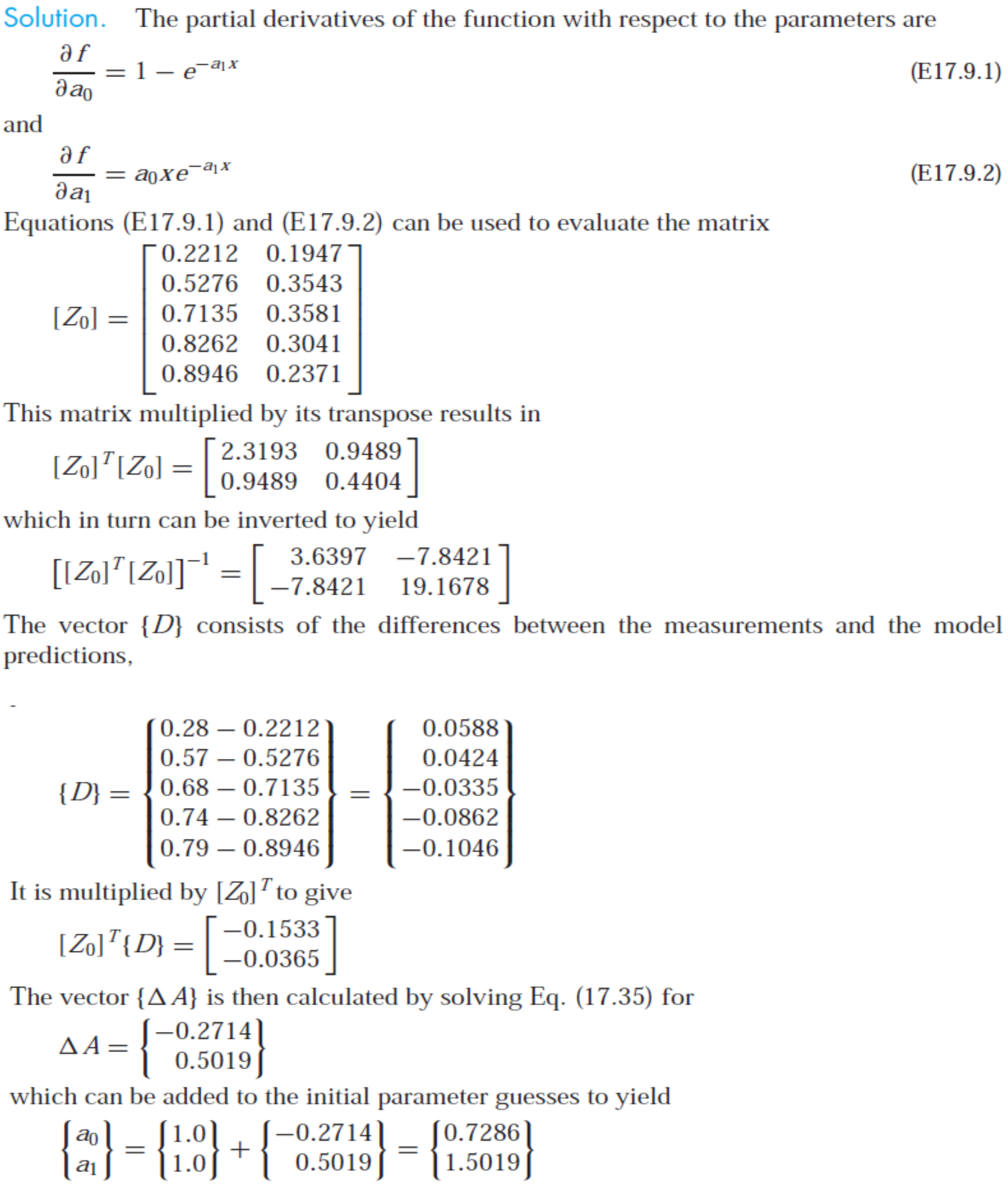
a=2.7557 b=0.4866 c=0.06095

$y=a+bx+cx^2$
$y=2.7557+0.4866x+0.06095x^2$
is the required parabola

c Fit the function $f(x;a_0,a_1)=a_0(1-e^{-a_1x})$ to the data:

x	0.25	0.75	1.25	1.75	2.25
y	0.28	0.57	0.68	0.74	0.79

using initial guesses $a_0=1$ *and* $a_1=1$. (Use Gauss Newton Method)



Solution. The partial derivatives of the function with respect to the parameters are

$$\frac{\partial f}{\partial a_0}=1-e^{-a_1x} \qquad \text{(E17.9.1)}$$

and

$$\frac{\partial f}{\partial a_1}=a_0xe^{-a_1x} \qquad \text{(E17.9.2)}$$

Equations (E17.9.1) and (E17.9.2) can be used to evaluate the matrix

$$[Z_0]=\begin{bmatrix}0.2212 & 0.1947\\ 0.5276 & 0.3543\\ 0.7135 & 0.3581\\ 0.8262 & 0.3041\\ 0.8946 & 0.2371\end{bmatrix}$$

This matrix multiplied by its transpose results in

$$[Z_0]^T[Z_0]=\begin{bmatrix}2.3193 & 0.9489\\ 0.9489 & 0.4404\end{bmatrix}$$

which in turn can be inverted to yield

$$[[Z_0]^T[Z_0]]^{-1}=\begin{bmatrix}3.6397 & -7.8421\\ -7.8421 & 19.1678\end{bmatrix}$$

The vector $\{D\}$ consists of the differences between the measurements and the model predictions,

-

$$\{D\}=\begin{Bmatrix}0.28-0.2212\\ 0.57-0.5276\\ 0.68-0.7135\\ 0.74-0.8262\\ 0.79-0.8946\end{Bmatrix}=\begin{Bmatrix}0.0588\\ 0.0424\\ -0.0335\\ -0.0862\\ -0.1046\end{Bmatrix}$$

It is multiplied by $[Z_0]^T$ to give

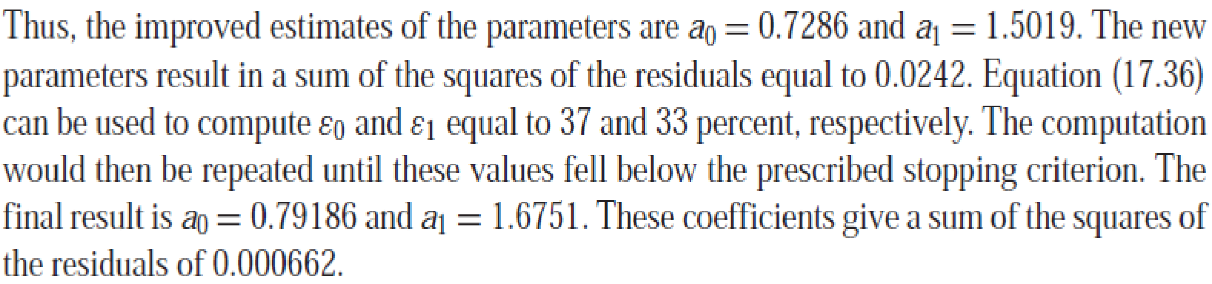
$$[Z_0]^T\{D\}=\begin{bmatrix}-0.1533\\ -0.0365\end{bmatrix}$$

The vector $\{\Delta A\}$ is then calculated by solving Eq. (17.35) for

$$\Delta A=\begin{Bmatrix}-0.2714\\ 0.5019\end{Bmatrix}$$

which can be added to the initial parameter guesses to yield

$$\begin{Bmatrix}a_0\\ a_1\end{Bmatrix}=\begin{Bmatrix}1.0\\ 1.0\end{Bmatrix}+\begin{Bmatrix}-0.2714\\ 0.5019\end{Bmatrix}=\begin{Bmatrix}0.7286\\ 1.5019\end{Bmatrix}$$



Thus, the improved estimates of the parameters are $a_0=0.7286$ and $a_1=1.5019$. The new parameters result in a sum of the squares of the residuals equal to 0.0242. Equation (17.36) can be used to compute ε_0 and ε_1 equal to 37 and 33 percent, respectively. The computation would then be repeated until these values fell below the prescribed stopping criterion. The final result is $a_0=0.79186$ and $a_1=1.6751$. These coefficients give a sum of the squares of the residuals of 0.000662.

d

Maximize $50x+100y$ subject to $10x+5y \le 2500; 4x+10y \le 2000; x+1.5y \le 450$ and $x \ge 0; y \ge 0$

A

Maximize Z= 50x + 100y
Subject to
10x+5y≤ 2500 …………(1)
4x+10y≤ 2000 …………(2)
X+1.5y≤ 450 …………(3)
And x≥ 0,y≥ 0
Convert the given constraints into equations 10x+5y= 2500 , 4x+10y= 2000 , X+1.5y= 450
Consider 10x+5y= 2500 *we get*

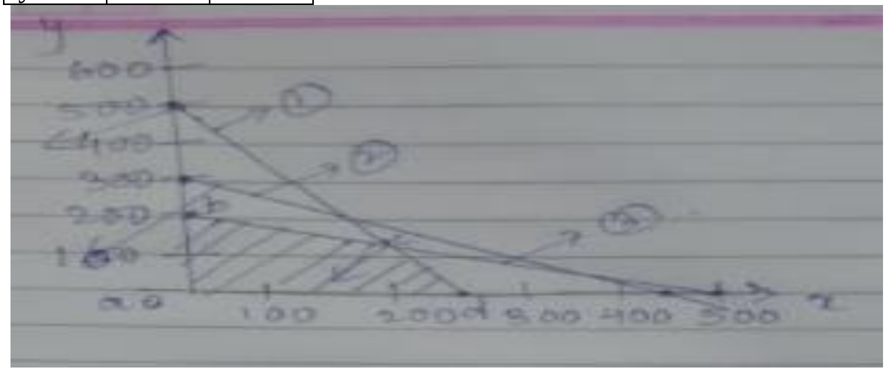
X	0	250
y	500	0

Consider 4x+10y= 2000 *we get*

X	0	500
y	200	0

Consider X+1.5y= 450 *we get*

X	0	450
y	300	0



Feasible region is ABCDA
A(0,0) ,B(0,200) ,C(200,110) D(250,0)

Points	Z=50x+100y
A	0
B	20000
C	21000
D	12500

Optimal Value of z is at C
X=200 and Y=110 is optimal solution

e

A firm makes two types of furniture – chairs and tables. The contribution for each product as calculated by the accounting department is Rs. 20 per chair and Rs. 30 per table. Both products are processed on three machines M_1 , M_2 and M_3 .The time required in hours by each product and total time available in hours per week on each machine are as follows:

MACHINE	CHAIR	TABLE	AVAILABLE TIME
M_1	3	3	36
M_2	5	2	50
M_3	2	6	60

How should the manufacturer schedule his production in order maximize contribution?

A

Maximize Z= 20x+30y
Subject to
3x+3y≤36
5x+2y≤50
2x+6y≤60
Non Negative constraints x≥ 0,y≥ 0

3x+3y≤36

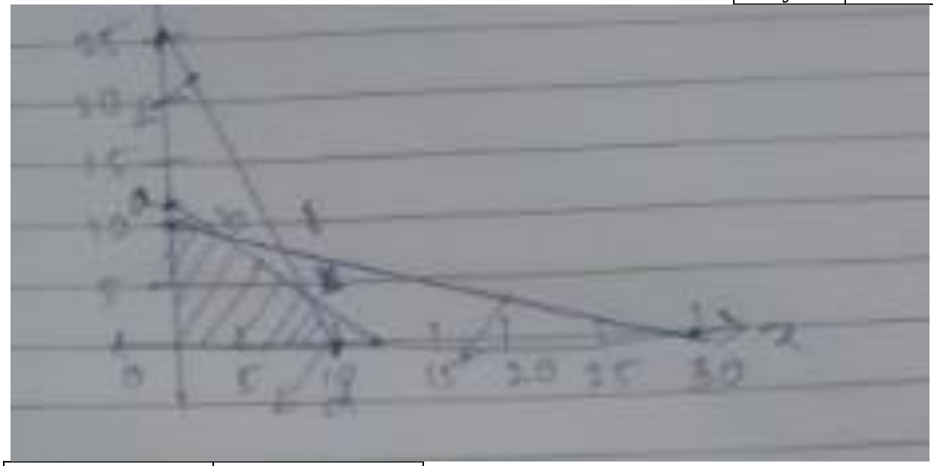
X	0	12
y	12	0

5x+2y≤50

X	0	10
y	25	0

2x+6y≤60

X	0	30
y	10	0



Vertex	Z=20x +30y
O=(0,0)	0
A=(0,10)	300
B=(5,8)	340
C=(4,5)	230
D=(10,0)	200

The maximum contribution come upto point b=(5,8) as x=5 and y=8

f An aged person must receive 4000 units of vitamin, 50 units of minerals and 1400 calories a day. A dietician advises to thrive on two foods F1 and F2 that cost Rs 4 and Rs 2 respectively per unit of food. It one unit of F1 contains 200 units of vitamins, 1 unit of mineral and 40 calories and one unit of F2 Contains 100 units of vitamins 2 units of minerals and 40 calories, formulate a linear programming model to minimize the cost of diet.

A

Product	Food F1	Food F2	Requirements
Vitamins	200	100	4000
Minerals	1	2	50
Calories	40	40	1400
Cost/Units	4	2	

Z=4x+2y

Subject to

200x+100y≥4000

x+2y≥50

40x+40≥1400

x,y≥0

5. **Attempt *any three* of the following:** **15**

a. The diameter of an electric cable; say X, is assumed to be a continuous random variable with p.d. f. $f\ (x\ )\ =\ 6x\ (1\ -\ x), 0\ \leq\ x\ \leq\ 1$.

(i) Check that above is p.d.f,

(ii) Determine a number b such that P (X <b) = P (X> b)

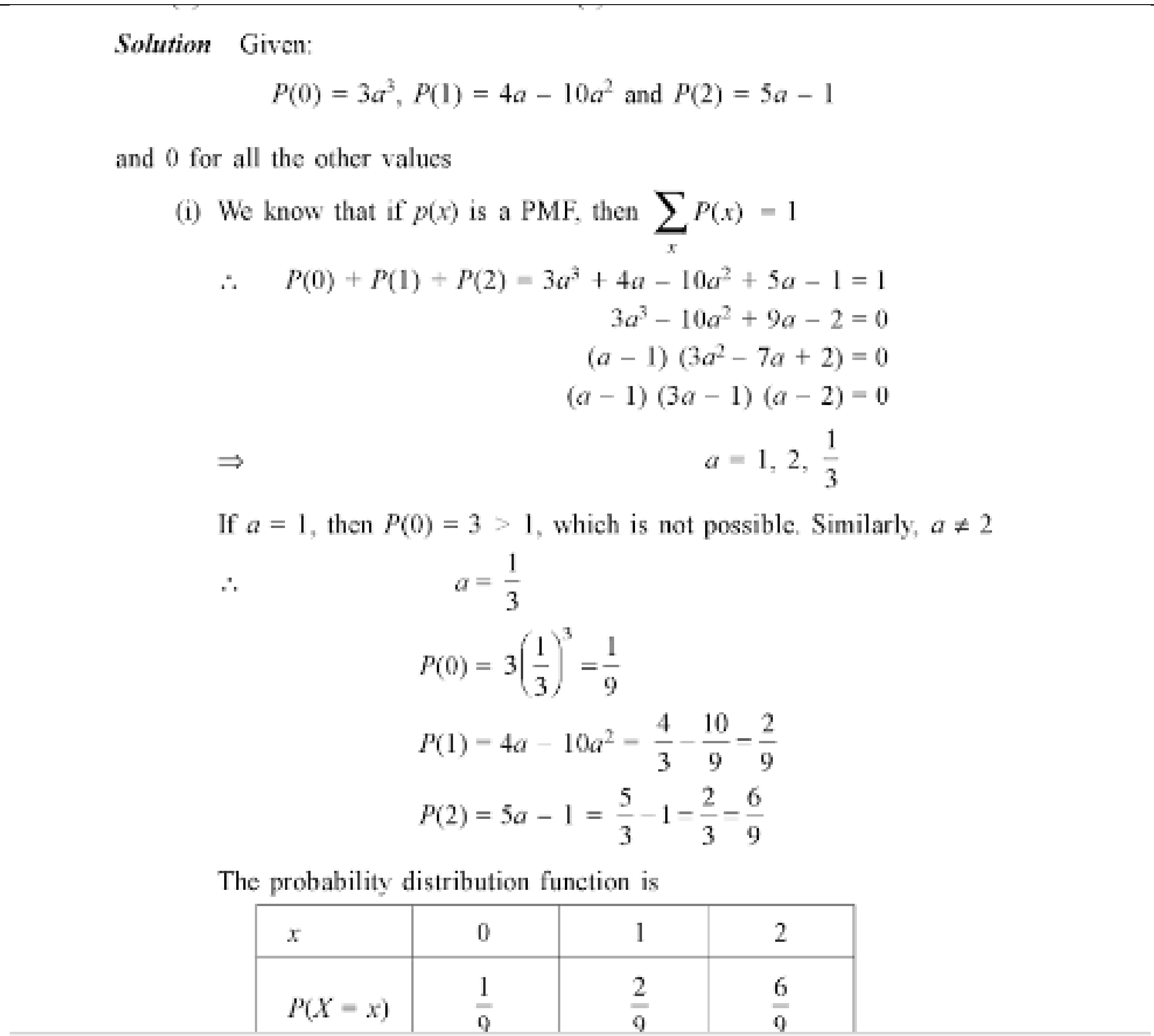
A

a) f(x) = 6x(1-x) , 0≤x≤1

(i) f(x)= 6x(1-x)

$\int_a^b f(x)dx = \int_0^1 6x(1-x)$dx

	= $\int_0^1 (6x - 6x^2)$dx = 1 ∴ pdf =1 f(x) is pdf. b) P(x < b) = Integral 0 to b ∫ 6x(1-x) dx P(x > b) = integral b to 1 ∫ 6x(1-x) dx ∫ 6x(1-x) dx =3x^2-2x^3 3b^2-2b^3 = [3(1)^2-2(1)^3 - 3b^2+2b^3] 3b^2-2b^3 = [1-3b^2+2b^3] 6b^2 -4b^3-1=0 b=1/2	
b.	Define and explain the concept of probability density function.	
A	Where the continuous probability distribution takes place called as probability distribution function. Let X be a continuous random variable. The function f|(X) is called the probability density function of x, if it satisfies the following i) f(x)≥ 0 x$\mathcal{E}\ R$ ii) $\int_{-\infty}^{\infty} f(x)\ dx$ Note : i) If x takes in a<x<b then the function f(x) is such that ii) $f(x) > 0\ for\ a < x < b$ iii) $\int_a^b f(x)\ dx$ = 1 iv) if (c,d) is an interval contained (a,b) then P(c<x<d) = $\int_c^d f(x) dx$ also P(c$\leq x \leq d$)=P(C<x$\sum d$) P(C$\leq x \leq d$) = $\int_c^d f(x) dx$ ∴ P(X=c) =0=P(X=d)	
c.	The probability mass function of a random variable X is zero except at the points $i = 0, 1{,}2$. At these points it has the values $p\ (0) = 3c^3$,$p(1) = 4c - 10c^2, p(2) = 5c - 1$ for some $c > 0$. *(i)* Determine the value of c. *(ii)* Compute the following probabilities,$P\ (X\ <\ 2)\ and\ P\ (1\ <\ X\ \leq\ 2)$. *(iii)* Describe the distribution function and draw its graph. *(iv)* Find the largest *x* such $thatF\ (x)\ < \frac{1}{2}$. *(v)* Find the smallest *x* such that$F\ (x) \geq \frac{1}{3}$.	
A		



Solution Given:

$$P(0) = 3a^3,\ P(1) = 4a - 10a^2 \text{ and } P(2) = 5a - 1$$

and 0 for all the other values

(i) We know that if $p(x)$ is a PMF, then $\sum_x P(x) = 1$

$$\therefore \quad P(0) + P(1) + P(2) = 3a^3 + 4a - 10a^2 + 5a - 1 = 1$$

$$3a^3 - 10a^2 + 9a - 2 = 0$$

$$(a - 1)(3a^2 - 7a + 2) = 0$$

$$(a - 1)(3a - 1)(a - 2) = 0$$

$$\Rightarrow \quad a = 1, 2, \frac{1}{3}$$

If $a = 1$, then $P(0) = 3 > 1$, which is not possible. Similarly, $a \neq 2$

$$\therefore \quad a = \frac{1}{3}$$

$$P(0) = 3\left(\frac{1}{3}\right)^3 = \frac{1}{9}$$

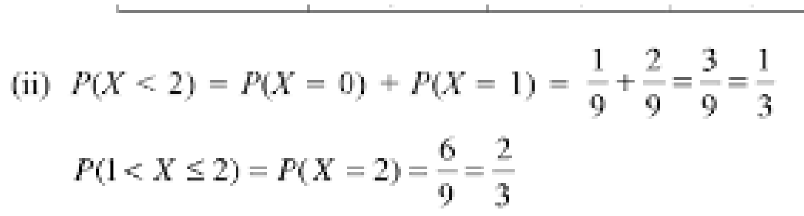
$$P(1) = 4a - 10a^2 = \frac{4}{3} - \frac{10}{9} = \frac{2}{9}$$

$$P(2) = 5a - 1 = \frac{5}{3} - 1 = \frac{2}{3} = \frac{6}{9}$$

The probability distribution function is

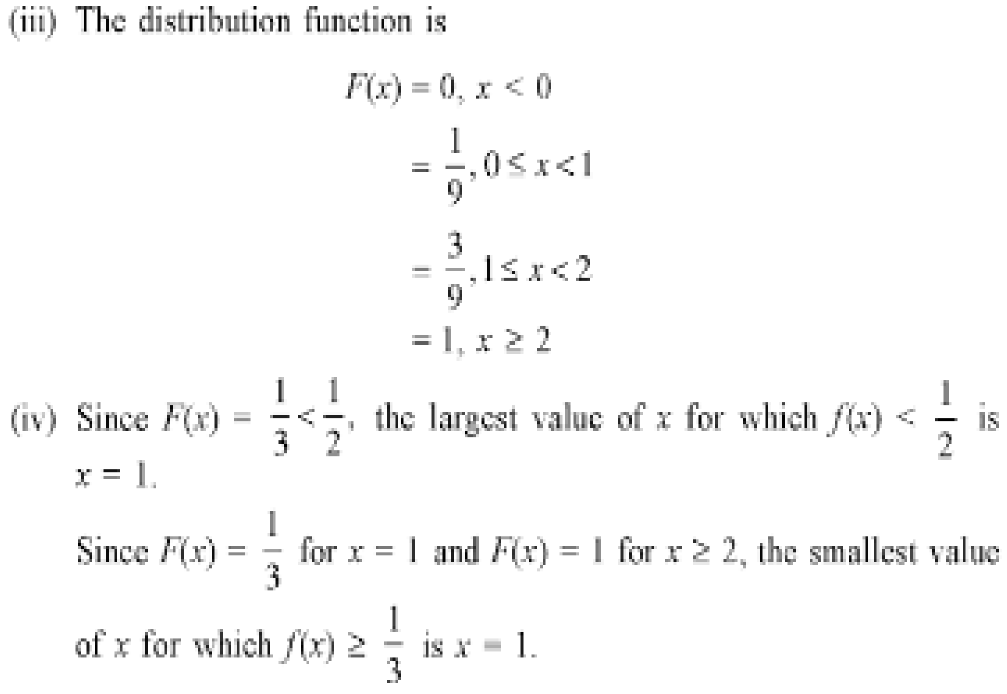
x	0	1	2
$P(X = x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{6}{9}$

** Consider **a** as **c**



(ii) $P(X < 2) = P(X = 0) + P(X = 1) = \frac{1}{9} + \frac{2}{9} = \frac{3}{9} = \frac{1}{3}$

$$P(1 < X \leq 2) = P(X = 2) = \frac{6}{9} = \frac{2}{3}$$



(iii) The distribution function is

$$F(x) = 0,\ x < 0$$
$$= \frac{1}{9}, 0 \le x < 1$$
$$= \frac{3}{9}, 1 \le x < 2$$
$$= 1,\ x \ge 2$$

(iv) Since $F(x) = \frac{1}{3} < \frac{1}{2}$, the largest value of x for which $f(x) < \frac{1}{2}$ is $x = 1$.

Since $F(x) = \frac{1}{3}$ for $x = 1$ and $F(x) = 1$ for $x \ge 2$, the smallest value of x for which $f(x) \ge \frac{1}{3}$ is $x = 1$.

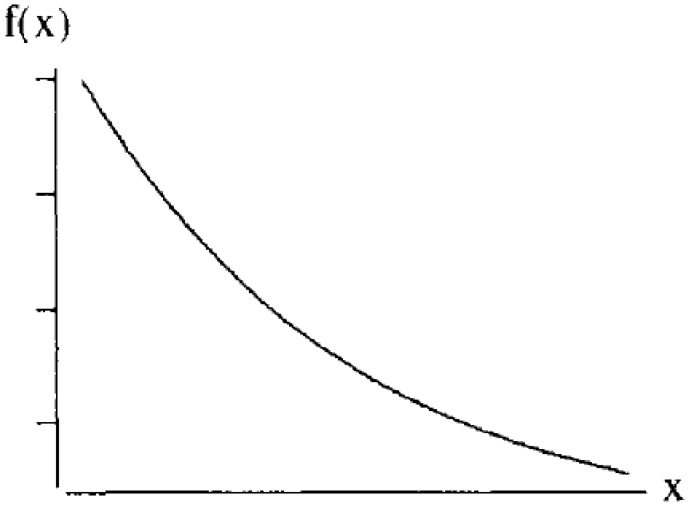
d. What is exponential distribution? Suppose the time till death after infection with Cancer, is exponentially distributed with mean equal to 8 years. If X represents the time till death after infection with Cancer, then find the percentage of people who die within five years after infection with Cancer.

A Exponential distribution:

The *exponential probability distribution* is a continuous probability distribution that is useful in describing the time it takes to complete some task. The pdf for an exponential probability distribution is given by formula below (where μ is the mean of the probability distribution and e = 2.71828 to five decimal places.

$$f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}} \quad for\ x \ge 0$$

The graph for the pdf of a typical exponential distribution is shown below:



The percentage of people who die within five years after infection with cancer is

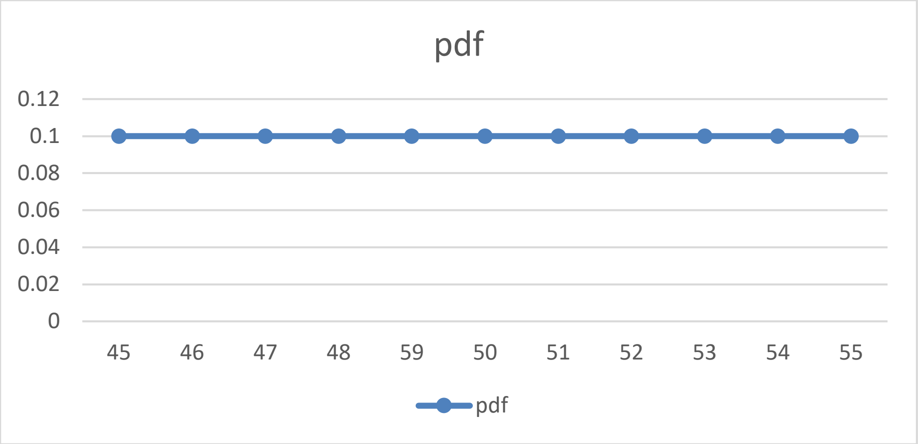
$$P(X \le 5) = 1 - e^{-\frac{5}{8}} = 1 - e^{-0.625} = 1 - 0.535 = 0.465\ i.e. 46.5\ \%$$

e. The price for a litre of whole milk is uniformly distributed between Rs. 45 and Rs. 55 during July in Mumbai. Give the equation and graph the pdf for X, the price per litre of whole milk during July. Also determine the percent of stores that charge more than Rs. 54 per litre.

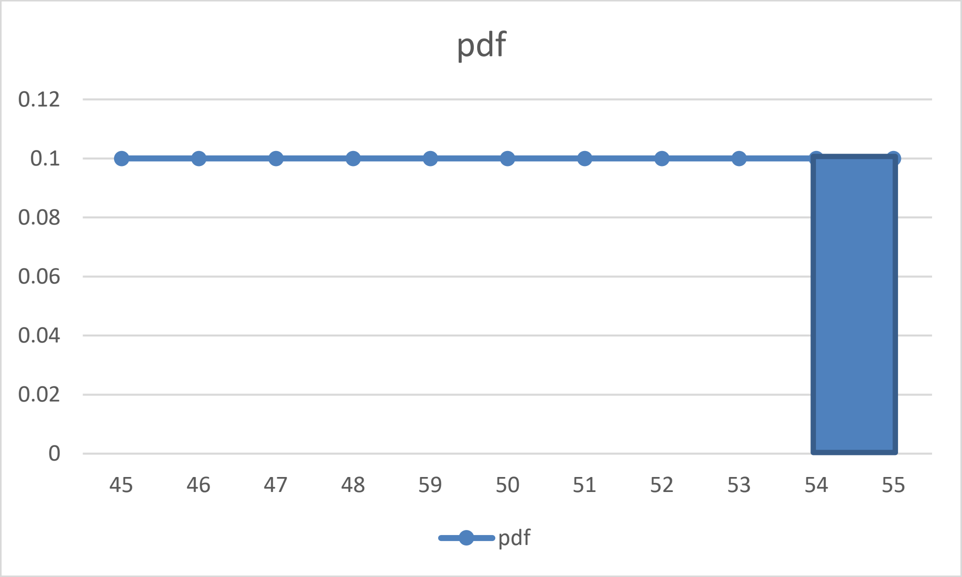
A The equation of pdf is

$$
\begin{aligned}
f(x) &= \frac{1}{55-45} = \frac{1}{10} = 0.1 && 45 < x < 55 \\
&= 0 && \text{elsewhere}
\end{aligned}
$$

The pdf is shown in the figure below:



The probability P(X>54) is shaded in the figure below:



The area of the shaded region is 0.1*1 = 0.1
Hence 0.10 * 100 = 10% of the stores charge more than Rs. 54/- per litre.

f. The monthly worldwide average number of airplane crashes of commercial airlines is 2.2.What is the probability that there will be (i)more than 2 such accidents in the next month? (ii) more than 4 such accidents in the next 2 months?

A

The number of crashes over a period of time is simply a random variable with a Poisson distribution. In this case, the su of two Poisson random variables is just a new random variable with the new rate being the sum of the old rates.

a. We have that $X_1 \sim \mathit{Poisson}(x_1;\ \lambda_1 = 2.2)$; this is just

$$\begin{aligned} P(X_1 > 2) &= 1 - P(X_1 \leq 2) \\ &= 1 - \left[e^{-2.2} + 2.2e^{-2.2} + \frac{(2.2)^2 e^{-2.2}}{2!} \right] \\ &= 0.3772 \end{aligned}$$

b. Let X_2 be the number of accidents that happen in a two month period. By additivity of the Poisson, $X_2 \sim \mathit{Poisson}(x_2;\ \lambda_2 = 2.2 + 2.2)$. Thus

$$\begin{aligned} P(X_2 > 4) &= 1 - P(X_2 \leq 4) \\ &= 0.44881619145568419 \end{aligned}$$